2013 IMSA Junior High Mathematics Competition

7^{th} Grade Individual

1. Compute the product of the roots of $(3x+1)(2x-5)=0$.

2. What is the difference between the sum of the first 1000 even integers and the sum of the first 1000 odd integers?

3. Compute the sum of the factors in 720.

4. What is the last digit of 148^{65342}?

5. The radii of two concentric circles are 10 and 26. What is the length of a chord of the larger circle that is externally tangent to the smaller circle?

6. Saieesh can write a math test in 2 hours. Kevin can write one in 3 hours. Jose can write one in 12 hours. Suppose that they work together to write a math test. How many hours will it take them to complete this daunting task? Express your answer as an improper fraction.

7. Dr. Condie has an unlimited number of 8 dollar bills, while Dr. Prince has an unlimited amount of 9 dollar bills. What is the largest amount of money (even dollar amount) that they cannot make?

8. Suppose the sides of a triangle have lengths 6, 8, and 10. What is the ratio of the triangle's circumradius to its inradius? Write your answer as a decimal.

9. How many indistinguishable ways are there to arrange the letters of THETA?

10. There are 200 students in the Class of 2014 at IMSA. If 135 like math, 101 like science, 34 like English, 47 like both math and science, 20 like both math and English, and 10 like both science and English. How many like all three subjects given that all students like at least one of the above subjects?

11. How many ways are there to arrange 4 keys on a keychain? Remember, rotations and flips matter.

12. Two numbers have a ratio of 3:4. If 4 is added to each number, the ratio is then 4:5. What is the value of the larger of the original two numbers?

13. If the average length of the bases of an isosceles trapezoid is 6 meters and the perimeter is 15 meters, what is the length of the leg? Express your answer as an improper fraction.

14. What is the smallest composite number that is not divisible by 2, 3, 5, or 7?

15. If $a-b=0$ and $ab=2$, what is the value of $\frac{a}{b}+\frac{b}{a}+2a^2+b^2$?

16. If 500! is multiplied out, how many trailing zeros are there?

17. The lengths of all sides of a right triangle are all integers. Suppose one side has length 13. Find the sum of all possible perimeters of all triangles that satisfy this scenario.

18. Determine the area of the region in the coordinate plane that is bounded by the lines $x=0, y=0, x=3$ and $y=4x+2$.

19. What is the value of $(1+2+3)+(2+3+4)+(3+4+5)+\cdots\cdots+(47+48+49)+(48+49+50)$?

20. How many perfect square factors does 18,000 have?

2013 IMSA Junior High Mathematics Competition

8^{th} Grade Individual

1. Given that the sum of the squares of two numbers is 42 and the product of the squares of the two numbers is 120, find the square of the sum of the two numbers.

2. A man is 5 times as old as his daughter. The sum of the squares of their ages is 2106. How old is the daughter?

3. Christine can wash windows in 10 minutes while Angela can wash the same amount of windows in 5 minutes. How many minutes will it take them to wash the windows if they work together? Express your answer as an improper fraction.

4. Imagine an 8×8 checkerboard. How many total squares (of any size) are there whose sides are coincident with the squares of checkerboard?

5. If a rectangle has perimeter 120, what is its maximum possible area, provided that the dimensions are integers and are not the same?

6. Suppose a regular hexagon and a square have the same perimeter. What is the ratio of the hexagon's area to the square's area? Express your answer in simple radical form.

7. Jennifer goes to Subway to get a sandwich. She can choose three toppings: meat, vegetables, and spread. She has 4 options for the meat (chicken, turkey, ham, salami), 3 options for vegetables (lettuce, tomatoes, onions), and 2 options for the spread (mustard, honey mustard). How many sandwiches can she possibly make given that she cannot make a sandwich with ham and tomatoes together?

8. The dimensions of a box are all integers and add up to 48. The box has a volume of 2800. What is the longest possible side of any such box?

9. John thinks of a two-digit number. When he reverses the digits, he realizes that the new number is 27 more than the original number. What is the sum of all possible two-digit numbers that John could have thought about?

10. Find the remainder when $(2^{20}+3^{30}+4^{40}+5^{50}+6^{60})$ is divided by 9.

11. What is the probability that I flip at least one tail when I toss 5 coins?

12. What is the largest possible value of n such that 2^n divides 500! ?

13. How many unique ordered triples of nonnegative integers (a,b,c) are there such that $a+b+c=9$?

14. If 4 cards are chosen from a standard deck of 52 cards without replacement. What is the probability that all 4 of the cards are of the same suit? Express your answer as a reduced fraction.

15. Find the least common multiple of 12, 20, 25, 32, and 150.

16. Find all integer solutions (x, y) of the system $x^2 + y = 12 = y^2 + x$. Write your answer as two ordered pairs.

17. What is the length of the apothem of a regular hexagon with a side length of 3 units? Express your answer in simple radical form.

18. Dr. Condie has an unlimited number of 8 dollar bills, while Dr. Prince has an unlimited amount of 9 dollar bills. What is the largest amount of money that they cannot make?

19. A committee of 4 people is chosen from a group of 5 women and 5 men. What is the probability that 2 men and 2 women will be chosen? Express your answer as a reduced fraction.

20. Find the number of ordered pairs of positive integers (x, y) such that $\frac{1}{x}+\frac{1}{y}=\frac{1}{7}$.

2013 IMSA Junior High Mathematics Competition

7^{th} Grade Team

1. Evaluate the cube root of 8^8.

2. What is the sum of the first 50 positive even positive integers?

3. Suppose we have a 100 pieces of candy. There are 20 red pieces, 30 blue pieces, 15 green pieces, 35 orange pieces. Each turn, I pick a candy. What is the minimum pieces of candy I need to pick to guarantee two pieces of the same color?

4. What percent of 75 is 30% of 150?

5. Compute the area of a regular hexagon with side length 4.

6. Calculate the area of an isosceles trapezoid which has bases 8 and 14 and has leg length of 5.

7. Compute the product of the divisors of 2013. Write your answer in the form 2013^x where x is an integer.

8. Skotlo lives on Mars and he forgot to go back to the bank, so he only has money in denominations of 7 and 9 dollar bills. Skotlo wants to buy N-game, a popular Mars video game. Given that the game costs the largest price not purchasable with only 7 and 9 dollar bills (without receiving change) and that Skotlo has a vast supply of these bills, find the price of the N-game.

9. Rohit is choosing what to wear today. He has 8 pairs of pants, 7 shirts, 3 hats, and 1 coat. He must wear exactly 1 pair of pants, 1 shirt, and 1 hat, but he can choose whether or not he wants to wear his coat. How many different combinations of outfits can he wear?

10. Given that $x + y = 7$ and $x^2 + y^2 = 17$, find $x^3 + y^3$.

11. Express .3244444….., (where the 4 is repeating) as a reduced fraction.

12. Suppose that a cube is inscribed in a sphere. If the sphere has surface area of 36π, find the volume of the cube. Express your answer as a simplified radical.

13. How many 5-digit palindromes are there? Note: a palindrome is a number of the form *abcba* where *a*,*b*,*c* are digits with $a \neq 0$, e.g. 25952.

14. Find the sum of all these 5-digit palindromes.

15. Abhishek is rowing his boat from his home up the Yellow River to a picnic area and then back home. It takes him 30 minutes to row up the river going against the current and 20 minutes to row back home with the current. Assume that Abhishek's rowing speed and the speed of the river current are constant. Suppose the rate that Abhishek rows is r miles per hour and the rate of the current is c miles per hour. Find $\frac{r}{c}$.

16. A chocolate box display is being set up in Jennifer's shop. The chocolate boxes are stacked in a pyramid with a square base and with one box in the top layer. Each box rests on the four boxes below it, except for the base. If the pyramid is to be 12 layers high, how many chocolate boxes are needed to construct it?

17. Find the largest 4-digit number such that the product of the digits is 432.

18. Eli the cow is shackled to a tree in the center of his owner's regular hexagonal yard by a 6-foot long chain. Given that the tree has a negligible size and the side length of the regular hexagonal yard is 4 feet, find the area, in square feet, outside the yard that Eli can still access. Express your answer in terms of π.

19. Find the area of the region that is enclosed by the graph of the equation $|3x-12|+|6y-42|=12$

20. Find the remainder when $f(x)=x^{81}+x^{32}+x^{6}+3x^{2}+1$ is divided by $x^{2}-1$. Express your answer in the form $ax+b$.

2013 IMSA Junior High Mathematics Competition

8th Grade Team

1. What is the degree measure of the acute angle between the hour hand and minute hand at 3:30 PM?

2. Suppose **$** is defined as the operation between two numbers *a* and *b*, where $a \,\$\, b = a^2 - b^2$. Calculate 53 **$** 47 + 96 **$** 94 + 22 **$** 18.

3. Draw a cyclic quadrilateral ABCD. Suppose that AB = 39, BC = 52, CD = 25, AD = 60. What is the radius of the circumscribed circle? Express your answer as a fraction.

4. How many ways are there to arrange 3 boys and 3 girls in a line given that no two people of the same gender can sit next to each other?

5. What is the radius of the inscribed circle of a triangle that has side lengths 13, 14, and 15?

6. A number is increasing if the digits of the number strictly increase from left to right. Therefore, 1234 is increasing but 1435 is not. How many 4-digit increasing numbers are there?

7. Suppose that it takes 4 days for 4 farmers to complete a job. How many days would it take 2 farmers to complete that same job?

8. How many ways are there to arrange 5 delegates around a table given that two delegates refuse to sit next to each other?

9. Draw square ABCD such that each side has length 4. Extend side DC to point E such that AE intersects BC at point M. If CE = 2, what is the length of BM? Express your answer as an improper fraction.

10. How many positive primes are there that are less than a 100?

11. What is the sum of all 5-digit palindromes? Note: a palindrome is a number of the form *abcba* where *a*, *b*, and *c* are digits with $a \neq 0$, e.g. 25952.

12. What is the units digit of the sum: 1! + 2! + 3! + 4! + 5! + …. + 10! + 11! + … + 100!?

13. A gardener plants 3 carrots, 4 onions, and 5 green peppers in a row. He plants them in random order, each arrangement being equally likely. Find the probability that no two green peppers are together. Express your answer as a reduced fraction.

14. Consider the sequence 1, 4, 5, 16, 17, 20, 21, 64, 65,.., which is formed by including only positive integers that can be expressed as the sum of distinct powers of 4. What is the 50th term in this sequence?

15. Let $n = 2^{12} \cdot 3^8 \cdot 5^6$. How many divisors of n^2 are less than *n*?

16. What angle (in degrees) does the line $y = \frac{2}{3}x - \frac{\pi}{4}$ make with the line $y = -\frac{3}{2}x - \frac{6}{7}$?

17. What is the value of $1-3+5-7+9-11+\cdots+97-99$?

18. I paint all faces of a $4\times4\times4$ cube. Then I cut the $4\times4\times4$ cube into 64 $1\times1\times1$ cubes. How many $1\times1\times1$ cubes have exactly two faces painted?

19. I have a group of 5 girls and 6 boys. How many ways are there for me to choose 2 girls and 2 boys for a 4-person committee?

20. Find the remainder when $f(x) = x^{81} + x^{32} + x^6 + 3x^2 + 1$ is divided by $x^2 - 1$?. Express your answer in the form $ax + b$.